Consolidity Analysis of Wind Turbines in Wind Farms

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Abstract- Consolidity is an inner property of systems, which explains the behavior of the stable and controllable systems operating in fully fuzzy conditions. Wind energy conversion systems are considered man-made fully fuzzy systems, which have been designed to achieve satisfied conditions such as stability, reliability, high efficiency, high performance and high accuracy. Sometimes these systems fail during its operation due to some mechanical or electrical issues or both. The authors in this paper present a novel consolidity analysis of the wind turbine rotor at different values of wind speed, pitch angle and tip speed ratio. This analysis shows how the consolidity trajectory pathways of the wind-turbine system changes from the unconsolidated zone to the consolidated zone and vice versa. It is foreseen that the consideration of the consolidity pathways will allow wind turbine designs that avoid failure of the operation of the wind-turbine. Consolidity charts that are constructed using the non-linear power coefficient formulas are used to show the behavior of wind turbines during external influences such as wind speed and internal influences such as blade pitch angle. The consolidity analysis presented in this paper incorporates theoretical data obtained from literature at rated values and practical data obtained from wind farms at Zafarana and the Gulf of El Zayt. The consolidity analysis shows that the degree of the consolidity of wind-turbine rotors falls in the unconsolidated class for the three case studies. The consolidity pathway is found to move towards consolidity zone while increasing both wind speed and blade pitch angle.

Keywords- Wind energy conversion systems; variable speed wind turbines; power coefficient of the wind turbines; consolidity index; consolidity charts; consolidity trajectory pathways; fully fuzzy systems.

1. Introduction

In the last few decades, the capacity of conventional fossil fuel resources decreased due to the increase of the demand and expand of human needs from electricity. Therefore renewable energy resources such as wind energy, solar energy, geothermal energy, biomass energy, wave energy and tidal energy became very attractive alternatives for the production of electricity, due to their numerous advantages such as being clean, as it reduced about 70 million metric tons of carbon dioxide emissions per year and very safe resources.Wind energy is the most attractive method to produce electricity, which have minimal running cost and environmentally friend energy source. Wind energy conversion systems WECS have very rapidly growth rate across the world, according to World Wind Energy Association, wind capacity reached by 597 GW in the end of 2018 [1-3].

According to the New & Renewable Energy Authority of Egypt (NREA), wind energy is currently preparing to participate 12% of total electricity production in Egypt at the end of the year 2022. The NREA has different wind projects locations, first location is Zafarana wind farm, South of Suez, Egypt, with average wind speed 9 m/s, and total rated capacity is 545 MW. Second location is the Gulf of El Zayt on the Gulf of Suez, Egypt, with average wind speed 10.5 m/s, and total rated capacity of 2850 MW at the end of year 2020 [4].

Wind turbines extract mechanical power from wind energy and convert this power into electrical power. The most common type of wind turbines are variable speed wind turbines (VSWTs).VSWTs have many merits such as achieving maximum mechanical power extracted from wind energy. Hence, they have high-energy conversion efficiency and maximum power point tracking (MPPT) achievement. The extracted mechanical power essentially relies on the power coefficient of the wind turbine $C_p(\beta, \lambda)$ [4, 5].

 $C_p(\beta, \lambda)$ depends on the design of wind turbine, defined by the three parameters wind speed V_w, tip speed ratio λ and blade pitch angle β .MPPT obtained when maintaining the VSWTs at optimal tip speed ratio λ_{opt} . It is achieved by driving the generator to the optimal speed ω_{opt} . The maximum power of a VSWT is produced by controlling the value of the blade pitch angle [5].

Following points represents the main concerns of the wind turbines manufacturers while designing VSWTs [1, 6, 7]:

a) Extraction of maximum mechanical power from wind energy.

b) The integration of the VSWTs into the electric power grid with constrained requirements of voltage and frequency.

c) Providing constant grid voltage and frequency.

d) Optimized performance of the VSWTs during mechanical disturbances such as varying of wind speed from minimum to maximum speed even in worst-case scenarios.

e) Overcome of the effects of the grid disturbances for various periods on the VSWTs.

f) The power performance of a wind turbine.

g) The stability, robustness and sensitivity of performance for the VSWTs during extreme conditions.

Although the above - mentioned design considerations are incorporated by the manufacturers, yet many events show collapse of VSWTs in wind farms.

In [8, 9]the authors introduced a detailed review of literature for the most common failures that occurs in wind turbine systems from the point of view of monitoring and maintenance of these failures. However, no effort was found in literature to explain the root cause of these failures. The authors in [10, 11] proposed an analysis for the systems consolidity using internal property of the system in what they defined as the consolidity theory. The behavior of any system is affected by the internal and/or external influences in inputs and system parameters, in fully fuzzy conditions.

In this paper, the authors are presenting a thorough novel analysis for the consolidity of the wind turbine rotor in an attempt to find an explanation for the failure that eventually occurs in VSWTs even though they are perfect designed for controllability, stability, reliability, etc.

In VSWTs, V_w , λ and β are the input variables, which affects the consolidity of the wind turbine rotor. Changes in these variables will be reflected in the consolidity index calculated.

Three case studies of the consolidity analysis reflecting the changes in the mechanical power P_m of the wind turbine for the changes of the input variables will be presented. Two different formulas of $C_p(\beta, \lambda)$ named here as sine formula [7] and exponential formula [1, 2] will be used. In each of these case studies, the below scenarios will be applied:

a) First scenario: wind speed, blade pitch angle and tip speed ratio are fuzzy parameters.

b) Second scenario: wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle.

First case represents the consolidity theoretical analysis of P_m using the optimal values of input variables from literature. The second case represents a practical consolidity analysis of the P_m using Egyptian Zafarana wind farm data through the year 2018/2019. The third case again represents a practical consolidity analysis but for the Egyptian Gulf of El-Zayt wind farm through the 2018/2019 year.

The consolidity analysis obtained from the three test cases was found to comply together where it is easily observed that the wind-turbine systems with their current designs are found to be unconsolidated in nature and the consolidity pathway trajectories indicate that it is directed towards consolidity whenever the wind speed and blade pitch angle are increased.

2. Consolidity Theorem

VSWTs are complex systems in real life, which affected by a lot of influences and parameters. VSWTs are stable systems by develop different control systems. Whatever they fail so a consolidity theory trying to check the hidden influences by using the new Arithmetic Fuzzy Logic- Based Representation approach. The new Arithmetic Fuzzy Logic relies on how calculating the fuzzy levels by its rules mentioned in literature [10, 11].

In particularly the consolidity theorem defined as the inner property of the fully fuzzy systems. The consolidity theorem depends on the value of the consolidity index. The consolidity index $\left| \frac{F_0}{F_{I+S}} \right|$ defined as positive ratio of the system overall output fuzziness F_0 behavior versus the combined input and systems parameters variations F_{I+S} , F_0 and F_{I+S} can be defined as follows [10, 11].

$$F_{I+S} = \frac{\sum_{i=1}^{m} V_{I_i} \cdot \ell_{I_i}}{\sum_{i=1}^{m} V_{I_i}} + \frac{\sum_{j=1}^{n} V_{S_j} \cdot \ell_{S_j}}{\sum_{j=1}^{n} V_{S_j}}$$
(1)

$$F_{0} = \frac{\sum_{i=1}^{k} V_{0_{i}} \cdot \ell_{0_{i}}}{\sum_{i=1}^{k} V_{0_{i}}}$$
(2)

So the consolidity index defined as positive ratio of

Consolidity index =
$$\left| \frac{F_0}{F_{1+S}} \right| = \left| F_{0/(l+S)} \right|$$
 (3)

where V_{I_i} , i=1, 2 ...m describe the deterministic value I_i and ℓ_{I_i} indicates its corresponding fuzzy level. V_{S_j} , j=1,2,...n, denote the deterministic value of system parameter S_j and ℓ_{S_j} denotes its corresponding fuzzy level. V_{O_i} , i=1, 2 ...k, designate the deterministic value of output component O_i and ℓ_{O_i} designates its corresponding fuzzy level.

the degree of consolidity of system depends on the value of consolidity index, which can be classified as follows:

- i. Consolidated system (C): for consolidity index < 1
- ii. Neutrally consolidated system (N): for consolidity index ≈ 1
- iii. Unconsolidated system (U): for consolidity index > 1

Consolidity chart construct for showing that changing of the system parameters in both inputs and system parameters. In addition, it shows how the output of the system affected during the changes of parameters below, on and above the references (setup points) of the system. Hence the consolidity pathway trajectory show the impacts of influences during the operation of the system from the unconsolidated zone to the consolidated zone and vice versa [10, 12]. Consolidity chart draw using mathematical method mentioned in [13].

3. Aerodynamic Model of the Wind Turbine

Wind turbines extract mechanical power from wind energy, in which the mechanical power can be expressed as [1, 5]:

$$P_{\rm m} = 0.5 \,\rho A V_{\rm w}^3 C_{\rm p}(\beta,\lambda) \tag{4}$$

where, ρ is the air density, A is the area swept by the rotor blades and equal to πR^2 , R is the radius of the turbine blade, V_w is the wind speed in m/s, and C_p(β , λ) is the power coefficient of a wind turbine or performance coefficient of a wind turbine. C_p(β , λ) is function of two variables which are the tip speed ratio λ and blade pitch angle β in degree.

Tip speed ratio can be defined as is the ratio of turbine speed at the tip of a blade to the free stream wind speed, given as [5]:

$$\lambda = \frac{R \,\omega_m}{V_w} = \frac{Rn\pi}{30V_w} \tag{5}$$

where, ω_m is the angular speed of the turbine in rad/sec and n is the rotational speed of the turbine in rpm.

 $C_p(\beta, \lambda)$ describes the relationship between the power that is captured by the wind turbine and the potential maximum power in the wind. This is a nonlinear function of β and λ . $C_p(\beta, \lambda)$ is calculated using variety of numerical approximation, empirical relationship and is validated in the laboratory. Two of the formulas used for the calculation power coefficient $C_p(\beta, \lambda)$ is introduced here and are named as sine formula and exponential formula [1, 5]. As mentioned before each of these two formulas will be examined for the consolidity analysis of the VSWT.

The so named here sine formula for the calculation of the power coefficient $C_p(\beta, \lambda)$ is given by [7]:

$$C_{p}(\beta,\lambda) = (0.44 - 0.0167\beta) \sin\left[\frac{\frac{\pi}{2}(\lambda-3)}{7.5 - 0.15\beta}\right] - (6)$$

0.00184 $\beta(\lambda - 3)$

The so named here exponential formula for the calculation of the power coefficient $C_p(\beta, \lambda)$ is given by [1, 2]:

$$C_{\rm p}(\beta,\lambda) = 0.5 \left(\begin{array}{c} \frac{98}{\lambda_{\rm i}} - 0.4\beta - 5 \right) \, e^{\left(\frac{-16.5}{\lambda_{\rm i}} \right)} \eqno(7)$$

with
$$\lambda_{i} = \left[\frac{1}{\lambda + 0.089} - \frac{0.035}{\beta^{3} + 1}\right]^{-1}$$
 (8)

4. Consolidity Analysis Applied to VSWTs

4.1. Case Study 1: Theoretical Analysis Using Optimal Values of Input Variables:

In this case, study the optimal values for β and λ will be utilized to examine the system consolidity.

4.1.1. Consolidity Analysis Using the Sine Formula

The optimal values of β and λ using the sine formula are $\beta = 0^{\circ}$ and $\lambda = 10.5$ [7]. In this case, study these optimal values for β and λ will be utilized to examine the system consolidity.

Substituting the optimal values of β and λ in "Eq. (6)", the maximum value of $C_p(\beta,\lambda)$ is found to 0.44 for rated $V_w = 13m/s$. Hence, maximum mechanical power extracted from wind occurred at these values.

4.1.1.1. First Scenario: Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

In this scenario VSWTs can be considered operating in fully fuzzy environment, assuming this system has V_w , β and λ as inputs fuzziness variables and mechanical power is output fuzziness, by using their optimal values, hence determining the output fuzzy level of the mechanical power. Substituting "Eq. (6)", in "Eq. (4)", then mechanical power will be given as:

$$P_{\rm m} = 0.44 \, {\rm QV}_{\rm w}^3 \, \sin\left[\frac{\pi/2 \, (\lambda-3)}{7.5 - 0.15\beta}\right] - \\ 0.0167\beta \, {\rm QV}_{\rm w}^3 \sin\left[\frac{\pi}{2}(\lambda-3)\right] - \\ 0.00184 \, \beta \, \lambda \, {\rm QV}_{\rm w}^3 + 0.00552\beta \, {\rm QV}_{\rm w}^3$$
(9)

where
$$Q = \frac{1}{2} \rho \pi R^2$$
, then let
 $X = 0.44 Q V_w^3 \sin \left[\frac{\pi/2 (\lambda - 3)}{7.5 - 0.15 \beta} \right]$ (10)

Y = 0.0167
$$\beta Q V_w^3 sin \left[\frac{\frac{\pi}{2} (\lambda - 3)}{7.5 - 0.15\beta} \right]$$
 (11)

$$Z = 0.00184 \,\beta \,\lambda \,Q \,V_w^3 \tag{12}$$

$$W = 0.00552 \,\beta \,Q \,V_{\rm w}^3 \tag{13}$$

Then "equation (9)" can be expressed as follows:

$$P_{\rm m} = X - Y - Z + W \tag{14}$$

By applying the new Arithmetic Fuzzy Logic-Based Representation approach which introduced in [10, 11] on "Eq. (14)", the fuzzy level of the mechanical output power can be calculated as follows:

$$L_{P_m} = \frac{1}{P_m} [XL_X - YL_{Y-}ZL_Z + WL_W]$$
(15)

where L_X , L_Y , L_Z and L_W are the fuzzy levels of variables X, Y, Z and W which calculated depends on the rules in [10, 11]:

$$L_{X} = 3L_{V_{W}} + \frac{\pi/2 (\lambda - 3)}{7.5 - 0.15\beta} \cot\left[\frac{\pi/2 (\lambda - 3)}{7.5 - 0.15\beta}\right] \left(\frac{\lambda L_{\lambda}}{(\lambda - 3)} + \frac{0.15\beta L_{\beta}}{7.5 - 0.15\beta}\right)$$
(16)

$$L_{Y} = L_{\beta} + 3L_{V_{W}} + \frac{\pi}{2} \left[\frac{\lambda L_{\lambda}}{7.5 - 0.15\beta} + \frac{0.15(\lambda - 3)\beta L_{\beta}}{(7.5 - 0.15\beta)^{2}} \right] \cot \left[\frac{\pi/2 (\lambda - 3)}{7.5 - 0.15\beta} \right]$$
(17)

$$L_{Z} = 3L_{V_{W}} + L_{\beta} + L_{\lambda} \tag{18}$$

$$L_{W} = L_{\beta} + 3L_{V_{W}} \tag{19}$$

Substituting "Eq. (8)" to "Eq. (13)" and "Eq. (16)" to "Eq. (19)" in "Eq. (15)" then the output fuzziness level of the mechanical power L_{P_m} can be calculated as follows:

$$L_{P_{m}} = \frac{1}{P_{m}} \left[0.44 Q V_{w}^{3} \sin N (3 L_{V_{w}} + K) - (20) \right]$$

$$0.0167 \beta Q V_{w}^{3} \sin N (3 L_{V_{w}} + L_{\beta} + K) - (20)$$

$$0.00184 \beta Q V_{w}^{3} \lambda (3 L_{V_{w}} + L_{\beta} + L_{\lambda}) + (20) \left[0.00552 \beta Q V_{w}^{3} (3 L_{V_{w}} + L_{\beta}) \right]$$

where

$$K = \frac{\pi}{2} \cot N \left(\frac{\lambda L_{\lambda}}{7.5 - 0.15\beta} + \frac{0.15 (\lambda - 3) \beta L_{\beta}}{(7.5 - 0.15\beta)^2} \right)$$
$$N = \frac{\pi/2 (\lambda - 3)}{7.5 - 0.15\beta}$$

 L_{V_w} , L_{λ} and L_{β} are the inputs fuzzy levels of V_w , λ and β . By using "Eq. (1)" to "Eq. (3)", the consolidity index of the mechanical power C_{P_m} is calculated as follows:

$$C_{P_{m}} = \left| \frac{F_{0}}{F_{I+S}} \right| = \left| \frac{L_{P_{m}}(V_{w} + \lambda + \beta)}{V_{w}L_{V_{W}} + \lambda L_{\lambda} + \beta L_{\beta}} \right|$$
(21)

By using optimal values of $(V_w, \lambda, \text{and }\beta)$ for the sine formula of the $C_p(\beta, \lambda)$. The consolidity indices obtained as shown in "Fig.1" indicate that the VSWTs will be an unconsolidated system as all points of the consolidity analysis are located in unconsolidated zone. The average consolidity index is equal to 10.54 which means that the degree of consolidity of the system is of the (U) class.



Fig. 1. Consolidity chart of C_{P_m} using Sine Formula, (Case Study 1, first scenario).

4.1.1.2. Second Scenario: Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle.

In this scenario, V_w and λ are fuzzy inputs parameters with their optimal values, and the mechanical power is output fuzzy parameter while using different deterministic values of β . This scenario is selected to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway. By applying the new Arithmetic Fuzzy Logic Representation, approach which is introduced in [9, 10] on "Eq. (10)" to "Eq. (13)", then the fuzzy levels of X, Y, Z and W for the second scenario can be calculated as follows [10, 11] :

$$L_{X} = 3L_{V_{W}} + K_{1} \lambda L_{\lambda} \cot[K_{1}(\lambda - 3)]$$
(22)

$$L_{Y} = 3L_{V_{W}} + K_{1} \lambda L_{\lambda} \cot[K_{1}(\lambda - 3)]$$
(23)

$$L_{Z} = 3L_{V_{W}} + L_{\lambda}$$
(24)

$$L_{W} = 3L_{V_{W}}$$
(25)

where
$$K_1 = \left[\frac{\pi/2}{7.5 - 0.15\beta}\right]$$

Substituting "Eq. (10)" to "Eq. (13)" and "Eq. (22)" to "Eq. (25)" in "Eq. (13)", then the output fuzziness level of the mechanical power L_{PM} can be calculated as follows:

$$L_{PM} = \frac{1}{PM} \left[0.44 Q V_w^3 \left(3 L_{V_w} + K_2 \right) \sin N - 0.0167 \beta Q V_w^3 \left(3 L_{V_w} + K_2 \right) \sin N - 0.00184 \beta Q V_w^3 \lambda \left(3 L_{V_w} + L_\lambda \right) + (26) 0.00552 \beta Q V_w^3 \left(3 L_{V_w} \right) \right]$$

where , $\,K_2=\frac{\pi}{2} \cot N\left(\frac{\lambda\,L_\lambda}{7.5-0.15\beta}\right)$

By using "Eq. (1)" to "Eq. (3)", the consolidity index of the mechanical power C_{P_m} is calculated as follows:

$$C_{PM} = \left| \frac{F_{O}}{F_{I+S}} \right| = \left| \frac{L_{PM}(V_{w} + \lambda)}{V_{w}L_{V_{W}} + \lambda L_{\lambda}} \right|$$
(27)

By assuming different deterministic values of β changing from 0° to 45° (theoretical), consolidity pathway behaviour can be examined. "Figure 2" shows how the VSWT consolidity pathway trajectory changes through the unconsolidated zone by changing the values of β . The smaller β , the closer the pathway trajectory to the neutral consolidated line. "Figure 3" indicates that the average consolidity index C_{P_m} slightly increases with the increase of β .



Fig. 2. Consolidity charts of C_{P_m} using Sine Formula, (Case Study 1, second scenario).



Fig. 3. Average consolidity index of C_{P_m} using Sine Formula, (Case Study 1, second scenario).

4.1.2. Consolidity Analysis Using the Exponential Formula

The optimal values of β and λ (using the exponential formula) are $\beta = 0^{\circ}$ and $\lambda = 8.1$ [14, 15]. In this case study this optimal values for β and λ will be utilized to examine the system consolidity.

Substituting these optimal values in "Eq. (7)", the maximum value of $C_p(\beta, \lambda)$ is found to 0.48 for rated wind speed V_w =12 m/s. Hence, maximum mechanical power extracted from wind occurs at these values.

4.1.2.1. First scenario: Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

By assuming, V_w , λ and β are inputs fuzzy parameters and mechanical power is the output fuzzy parameter. Hence determined the value of C_{P_m} as follows:

Substituting "Eq. (8)" in "Eq. (7)", and then substituting the result in "Eq. (4)", the mechanical power can be easily deduced to be given by:

$$P_{\rm M} = Q_1 V_{\rm w}^3 \left[\frac{98}{\lambda + 0.089} - \frac{3.43}{\beta^3 + 1} - 0.4\beta - \right] e^{\frac{-16.5}{\lambda + 0.089}} e^{\frac{0.5775}{\beta^3 + 1}}$$
(28)

where
$$Q_1 = \frac{1}{4} \rho \pi R^2$$
, then let
 $A = V_w^3$
(29)

$$B = \left[\frac{98}{\lambda + 0.089} - \frac{3.43}{\beta^3 + 1} - 0.4\beta - 5\right]$$
(30)

$$C = e^{\frac{-16.5}{\lambda + 0.089}}$$
(31)

$$D = e^{\frac{0.5775}{\beta^3 + 1}}$$
(32)

Using "Eq. (29)" to (32), "Eq. (28)" can be expressed as:

$$PM = Q_1 ABCD \tag{33}$$

By applying the new Arithmetic Fuzzy Logic-Based Representation approach which is given in [10, 11] on "Eq. (33)" .So the fuzzy level of the mechanical output power can be calculated as follows:

$$L_{P_M} = L_A + L_B + L_C + L_D \tag{34}$$

where L_{A} , L_{B} , L_{C} and L_{D} are the fuzzy levels of variables A,

B, C and D which calculated using rules in [10, 11] as follows:

$$L_{A} = 3L_{V_{W}}$$
(35)

$$L_{B} = \frac{1}{B} \left[\frac{10.29\beta^{3} L_{\beta}}{(\beta^{3}+1)^{2}} - \frac{98\lambda L_{\lambda}}{(\lambda+0.089)^{2}} - 0.4\beta L_{\beta} \right]$$
(36)

$$L_{\rm C} = \frac{16.5\,\lambda L_{\lambda}}{(\lambda + 0.089)^2} \tag{37}$$

$$L_{\rm D} = \frac{-1.7325\,\beta^3\,L_{\beta}}{(\beta^3 + 1)^2} \tag{38}$$

Substituting by "Eq. (35)" to "Eq. (38)" in "Eq. (34)" so the output fuzziness level of the mechanical power can be calculated as follows:

$$L_{PM} = \frac{1}{B} \left(\frac{10.29\beta^{3}L_{\beta}}{(\beta^{3}+1)^{2}} - \frac{98\lambda L_{\lambda}}{(\lambda+0.089)^{2}} - 0.4\beta L_{\beta} \right) + \frac{16.5\lambda L_{\lambda}}{(\lambda+0.089)^{2}} - \frac{1.7325\beta^{3}L_{\beta}}{(\beta^{3}+1)^{2}} + 3L_{V_{W}}$$
(39)

The consolidity index C_{P_m} is calculated using "Eq. (21)", considering the optimal values of $(V_w, \lambda, \text{and }\beta)$ for the exponential formula of $C_p(\beta, \lambda)$. As indicated in "Fig. 4", the VSWTs will be unconsolidated system (all points of consolidity analysis are located in unconsolidated zone), as the average consolidity index is equal to 11.41 as shown in "Fig. 4", the degree of the system is considered of the (U) class.



Fig. 4. Consolidity chart of C_{P_m} using Exp Formula, (Case Study 1, first scenario).

4.1.2.2. Second scenario: Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle

In this scenario, V_w and λ are fuzzy inputs parameters with their optimal values, and the mechanical power is output fuzzy parameter while using different deterministic values of β . This scenario is selected to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway of VSWTs.

"Eq. (28)" can be rewritten as follows: $P_M = Q_2 ABC$ (40)

where, $Q_2 = \frac{1}{4} \rho \pi R^2 e^{\frac{0.5775}{\beta^3+1}}$, by applying the new Arithmetic Fuzzy Logic-Based representation approach which introduced in [9, 10] on "Eq. (40)". The fuzzy level of the mechanical output power can be calculated as:

$$L_{P_M} = L_A + L_B + L_C \tag{41}$$

 L_A and L_C has been deduced "Eq. (35)" and "Eq. (37)" respectively, then calculation of the L_B using the rules in [10, 11] with the consideration that V_w and λ are fuzzy variables will be given as:

$$L_{\rm B} = \frac{1}{\rm B} \left[\frac{-98\,\lambda\,L_{\lambda}}{(\lambda + 0.089)^2} \right] \tag{42}$$

Substituting "Eq. (35)", "Eq. (37)" and "Eq. (42)" in "Eq. (41)", the output fuzziness level of the mechanical power can be calculated as follows:

$$L_{P_{M}} = 3L_{V_{W}} + \frac{1}{B} \left(\frac{-98\lambda L_{\lambda}}{(\lambda + 0.089)^{2}} \right) + \frac{16.5\lambda L_{\lambda}}{(\lambda + 0.089)^{2}}$$
(43)

The C_{P_m} can be calculated using "Eq. (27)".

It can be easily observed from "Fig. 5" that the consolidity indices are located in unconsolidated zone. Also the consolidity pathway changes through the unconsolidated zone by changing the values of β from high value (11.5) of C_{Pm} to low value (10.9) of C_{Pm} while increasing deterministic values of the β from 10° till 30° (theoretical). As seen in "Fig. 6" average of C_{Pm} decreasing with increasing deterministic values of the β from 0° till 45° (theoretical).



Fig. 5. Consolidity chart of C_{P_m} using Exp Formula, (Case Study 1, second scenario).



Fig. 6. Average consolidity region of C_{P_m} using Exp Formula, (Case Study 1, second scenario).

4.2. Case Study 2: Zafarana Wind Farm

NREA is the authority, which is responsible for new and renewable energy such as wind projects in Egypt. Zafarana wind farm is one of oldest wind projects in Egypt, which connected to electric grid in 2001. With total rated output capacity 545 MW, with average wind speed 9 m/s. This wind farm contains 700 (VSWTs) with different types of rated output power (600 kW, 660 kW and 850 kW). Consolidity studies made using different production data of Zafarana wind farm through the year 2018/2019. By using different data for different rated wind turbines with different locations and conditions. As seen in case study 1 of the consolidity analysis for theoretical optimal values of V_w , λ and β . The analysis investigated for input variables (V_w , λ and β) in practical Zafarana wind farm data.Hence C_{P_m} can be calculated and drawing using consolidity charts in Matlab/m-files.

4.2.1. Consolidity Analysis Using the Sine Formula

4.2.1.1. First scenario. Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

In this scenario V_w , λ , β of practical data considered inputs fuzzy parameters. Then the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (20)", which considered

as the output fuzzy parameter. After that using "Eq. (21)" to calculate the $C_{P_{\rm m}}$.

As shown in "Fig. 7", the consolidity chart of VSWTs is located in unconsolidated zone and all practical points located at unconsolidated zone. Therefore, the degree of the system is considered of the (U) class.

As shown in "Fig. 8", the value of C_{P_m} decrease from 24.45 at $V_w = 9$ m/s to 4.14 at $V_w = 15$ m/s. Also in "Fig. 8", the annual average value of β increase with increasing the annual average of V_w , as one of variables control of VSWTs stability. Furthermore the value of $\beta = 1.94^{\circ}$ at $V_w = 9$ m/s and the value of $\beta = 10.72^{\circ}$ at $V_w = 15$ m/s. Finally, C_{P_m} of the VSWTs decreases with the increase of both wind speed and blade pitch angle.



Fig. 7. Consolidity chart of C_{Pm} using Sine Formula, (Case Study 2, first scenario).



Fig. 8. Values of C_{P_m} versus β and V_w using Sine Formula, (Case Study 2, first scenario).

4.2.1.2. Second scenario. Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle

In this scenario V_w and λ of practical data considered inputs fuzzy parameters while using different deterministic values of β from practical data used to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway of VSWTs. The consolidity pathway is drawing by consolidity charts. Furthermore, the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (26)", which considered as the output fuzzy parameter. After that using "Eq. (27)" to calculate the C_{P_m} .

As shown in "Fig. 9", all points of consolidity analysis are located in unconsolidated zone. The consolidity pathway of the VSWTs decrease from high value of average C_{P_m} 10.18 with deterministic value of β is 1.94° to low value of average C_{P_m} 3.96 with deterministic value of β is 10.72°. Therefore, the average C_{P_m} decrease with increasing of β .

As shown in "Fig. 10", the value of β increased with the increase of V_w . Hence, C_{P_m} of the VSWTs decreases with the increase of both wind speed and blade pitch angle.



Fig. 9. Consolidity charts of C_{P_m} using Sine Formula, (Case Study 2, second scenario).



Fig. 10. Values of C_{P_m} versus β and V_w using Sine Formula, (Case Study 2, second scenario).

4.2.2. Consolidity Analysis using the Exponential Formula

4.2.2.1. First scenario. Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

In this scenario V_w , λ , β of practical data considered inputs fuzzy parameters. Then the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (39)", which considered as the output fuzzy parameter. After that using "Eq. (21)" to calculate the C_{P_m} .

As shown in "Fig. 11", the consolidity chart of VSWTs is located in unconsolidated zone and the consolidity indices obtained from practical points located at unconsolidated zone. Therefore, the degree of the system is considered of the (U) class.Also as shown in "Fig. 12", the value of C_{P_m} decrease from 16.38 at $V_w = 9$ m/s and $\beta = 1.94^\circ$ to 4.88 at $V_w = 15$ m/s and $\beta = 10.72^\circ$. Finally, the value of C_{P_m} decreases with the increase of both wind speed and blade pitch angle.



Fig. 11. Consolidity chart of C_{P_m} using Exp Formula, (Case Study 2, first scenario).



Fig.12. Values of C_{P_m} versus β and V_w using Exp Formula, (Case Study 2, first scenario).

4.2.2.2. Second scenario. Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle

In this scenario V_w and λ of practical data considered inputs fuzzy parameters while using different deterministic values of β from practical data used to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway of VSWTs. The consolidity pathway is drawing by consolidity chart using different values of β . Furthermore, the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (43)", which considered as the output fuzzy parameter. After that using "Eq. (27)" to calculate the C_{P_m} .

As shown in "Fig.13", all points of consolidity analysis are located in unconsolidated zone. The consolidity pathway of the VSWTs decrease from high value of average C_{P_m} is

30.15 with deterministic value of β is 1.94° to low value C_{P_m} is 4.28 with deterministic value of β is 10.72°. Therefore, the average C_{P_m} decrease with increasing of β .

As shown in "Fig.14", the value of β increase with increasing V_w.Hence, the average C_{P_m} decreases with the increase of V_w and β .



Fig. 13. Consolidity charts of C_{P_m} using Exp Formula, (Case Study 2, second scenario).



Fig. 14. Values of C_{P_m} versus β and V_w using Exp Formula, (Case Study 2, second scenario).

The summary results of Zafarana wind farm are shown in "Table 1.", "Table 2."

Table 1. Summary results of first scenario of Zafarana wind farm.

Formula of $C_p(\beta, \lambda)$	Sine Formula	Exponential Formula
Class of Consolidity	unconsolidated	unconsolidated
Average of C_{P_m}	decrease	decrease
Average of C_{P_m} versus β	decrease from 24.45 to 4.14 with increase β from (1.94° to 10.72°) respectively	decrease from 16.38 to 4.88 with increase β from (1.94° to 10.72°) respectively

Average of C _{Pm} versus V _w	decrease from 24.45 to 4.14 with increase V_w from (9 m/s to 15 m/s) respectively	decrease from 16.38 to 4.88 with increase V_w from (9 m/s to 15 m/s) respectively
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 Table 2. Summary results of second scenario of Zafarana wind farm.

Formula of $C_p(\beta, \lambda)$	Sine Formula	Exponential formula
Class of Consolidity	unconsolidated	unconsolidated
Average of C _{Pm}	decrease	decrease
Average of C_{P_m} versus β	decrease from 10.18 to 3.96 with increase β from (1.94° to 10.72°) respectively	decrease from 30.15 to 4.28 with increase β from (1.94° to 10.72°) respectively
Average of C_{P_m} versus V_w	decrease from 10.18 to 3.96 with increase V_w from (9 m/s to 15 m/s) respectively	decrease from 30.15 to 4.28 with increase V_w from (9 m/s to 15 m/s) respectively

4.3. Case Study 3: The Gulf of El Zayt Wind Farm

The Gulf of El Zayt wind farm is located at the red sea area with average wind speed 10.5 m/s over the year. The total power capacity of this wind farm it will be about 3000 MW at the end of year 2024[4].NREA planned to increase the electricity produced from the wind energy to 14% of the total production electricity in Egypt at the end of 2035. By planned projects under development in the Gulf of the Suez and West of the Nile with total capacity 1970 MW according to the 2018 NREA report.

Consolidity studies made using different production data of the Gulf of El Zayt wind farm through the year 2018/2019. The rated power of the VSWTs in The Gulf of El Zayt is 2 MW. The study investigated for V_w , λ and β as input fuzzy parameters in practical Gulf of El Zayt wind farm data. Hence the C_{P_m} can be calculated and drawing the consolidity charts and consolidity pathways trajectory during this period of the study.

4.3.1. Consolidity Analysis Using the Sine Formula

4.3.1.1. First scenario. Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

In this scenario V_w , λ , β of practical data considered input fuzzy parameters. Then the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (20)", which considered as the

output fuzzy parameter. After that using "Eq. (21)" to calculate the C_{P_m} .

As shown in "Fig.15", the consolidity chart of VSWTs is located in unconsolidated zone and the consolidity indices obtained from practical points fall in the unconsolidated zone. Therefore, the degree of the system is considered of the (U) class.

As shown in "Fig. 16", the value of C_{P_m} decrease from 20.96 at $V_w = 8.6$ m/s to 4.35 at $V_w = 17$ m/s. In addition, the annual average value of β increase while increasing the annual average of V_w , as one of the variables that control the VSWTs stability. Also in "Fig. 16", the value of $\beta = -0.97^\circ$ at $V_w = 8.6$ m/s and the value of $\beta = 10.32^\circ$ at $V_w = 17$ m/s. Finally, the value of C_{P_m} of the system is decreasing with the increase of both wind speed and blade pitch angle.



Fig.15. Consolidity chart of C_{P_m} using Sine Formula, (Case Study 3, first scenario).



Fig.16. Values of C_{P_m} versus β and V_w using Sine Formula, (Case Study 3, first scenario).

4.3.1.2. Second scenario. Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle

In this scenario V_w and λ of practical data considered inputs fuzzy parameters while using different deterministic values of β from practical data used to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway of VSWTs. The consolidity pathway is drawn using

the consolidity chart for different values of β . Furthermore, the fuzzy level of mechanical power L_{P_M} , which is the output fuzzy parameter, is calculated using "Eq. (26)". After that using "Eq. (27)" to calculate the C_{P_m} .

As shown in "Fig. 17", all points of consolidity analysis are located in unconsolidated zone. The consolidity pathway of the VSWTs decreases from the highest value of average C_{P_m} (20.96), with deterministic value of β is -1.002° to the lowest value of average C_{P_m} (1.96), with deterministic value of β is 10.81°. Hence C_{P_m} decreases with the increase of β . As shown in "Fig.18", the value of β increase with increasing V_w .Hence C_{P_m} decreases with the increase of both V_w and β .



Fig. 17. Consolidity charts of C_{P_m} using Sine Formula, (Case Study 3, second scenario).



Fig.18. Values of C_{P_m} versus β and V_w using Sine Formula, (Case Study 3, second scenario).

4.3.2. Consolidity Analysis using the Exponential Formula

4.3.2.1. First scenario. Wind speed, blade pitch angle and tip speed ratio are fuzzy parameters

In this scenario V_w , λ , β of practical data considered inputs fuzzy parameters. Then the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (39)", which considered as the output fuzzy parameter. After that using "Eq. (21)" to calculate the C_{P_m} .

As shown in "Fig.19", the consolidity chart of VSWTs is located in unconsolidated zone and the consolidity indices obtained from practical points located at unconsolidated zone. Therefore, the degree of the system is considered of the (U) class.

As shown in "Fig.20", the value of C_{P_m} decrease from 13.36 at $V_w = 8.6$ m/s to 2.85 at $V_w = 17$ m/s. In addition, the annual average value of β increase with increasing the annual average of V_w . Also in "Fig. 20", the value of $\beta = -0.97^\circ$ at $V_w = 8.6$ m/s and the value of $\beta = 10.32^\circ$ at $V_w = 17$ m/s. Finally, the value of C_{P_m} of the system decreases with the increase of both V_w and β .



Fig.19. Consolidity chart of C_{P_m} using Exp Formula, (Case Study 3, first scenario).



Fig.20. Values of C_{P_m} versus β and V_w using Exp Formula, (Case Study 3, first scenario).

4.3.2.2. Second scenario. Wind speed and tip speed ratio are fuzzy parameters, with different deterministic values of blade pitch angle

In this scenario V_w and λ of practical data considered inputs fuzzy parameters while using different deterministic values of β from practical data used to examine the effect of β on the average value of C_{P_m} and the change of the consolidity pathway of VSWTs.Furthermore, the fuzzy level of mechanical power L_{P_M} calculated by using "Eq. (43)", which considered as the output fuzzy parameter. After that using "Eq. (27)" to calculate the C_{P_m} .

As shown in "Fig.21", all points of consolidity analysis are located in unconsolidated zone. The consolidity pathway of the VSWTs decrease from high value of average C_{P_m} is 27.68 with deterministic value of β is -1.002° to low value of C_{P_m} is 6.69 with deterministic value of β is 10.81°. Therefore, the average C_{P_m} decrease with increasing of β .

As shown in "Fig.22", the value of β increase with increasing V_w . Hence C_{P_m} decreases with the increase of both V_w and β .



Fig. 21. Consolidity charts of C_{P_m} using Exp Formula, (Case Study 3, second scenario).



Fig. 22. Values of C_{P_m} versus β and V_w using Exp Formula, (Case Study 3, second scenario).

The summary results of Zafarana wind farm are shown in "Table 3.", "Table 4."

Table 3. Summary results of first scenario of The Gulf of ElZayt Wind Farm

Formula of $C_p(\beta, \lambda)$	Sine Formula	Exponential Formula
Class of Consolidity	unconsolidated	unconsolidated
Average of C_{P_m}	decrease	decrease
Average of C_{P_m} versus β	decrease from 20.96 to 4.35 with increase β	decrease from 13.36 to 2.85 with increase β from (-0.97° to

	from (-0.97° to 10.32°) respectively	10.32°) respectively
Average of C _{Pm} versus V _w	decrease from 20.96 to 4.35 with increase V_w from (8.6 m/s to 17 m/s) respectively	decrease from 13.36 to 2.85 with increase V_w from (8.6 m/s to 17 m/s) respectively

Table 4. Summary results of second scenario of the Gulf of ElZayt wind farm.

Formula of $C_p(\beta, \lambda)$	Sine Formula	Exponential formula
Class of Consolidity	unconsolidated	unconsolidated
Average of C_{P_m}	decrease	decrease
Average of C_{P_m} versus β	decrease from 9.99 to 1.96 with increase β from (-1.002° to 10.81°) respectively	decrease from 27.68 to 6.69 with increase β from (-1.002° to 10.81°) respectively
Average of C_{P_m} versus V_w	decrease from 9.99 to 1.96 with increase V_w from (8.4 m/s to 17.3 m/s) respectively	decrease from 27.68 to 6.69 with increase V_w from (8.4 m/s to 17.3 m/s) respectively

5. Discussion

VSWTs are multiple inputs and outputs, nonlinear systems. The first case study shows that the degree of C_{P_m} is unconsolidated for both Sine and Exponential formula and for the two scenario. In the second scenario, different deterministic values of β were applied at constant optimal V_w and λ . As shown in "Fig. 3", the average C_{P_m} is increasing for Sine formula. Nevertheless, for the Exponential formula the average C_{P_m} is decreasing as shown in "Fig. 6". This can be explained in the light of using of the theoretical data for changes in the value of β while considering constant wind speed V_w and tip speed ratio λ , which is not the case from a practical point of view.

For second and third case, numerous practical values of β and λ related to the same V_w obtained from the SCADA system. So the annual average of β and λ at each specific V_w is used for drawing the C_{P_m} through the period of the study as shown in "Fig. 8", "Fig. 9", "Fig. 12", "Fig. 14", "Fig. 16", "Fig. 18", "Fig. 20" and "Fig. 22". In the second scenario, the practical values of β used as deterministic values related to changeable values of V_w and λ . Hence, C_{P_m} is decreasing for both Sine and Exponential formula. Furthermore, the degree

of consolidity of VSWTs is unconsolidated class for both Sine and Exponential formula for the two scenario.

 L_{V_w}, L_{λ} and L_{β} are inputs fuzzy levels of V_w, λ and β which were assumed to have different values using random function in Matlab in order to prove the degree of consolidity of VSWTs. In all cases, the same results of the consolidity analysis are obtained.

It is worth also to mention that in real wind farms, the range of the β falls between -3° and 90° depending on the state of VSWTs.

6. Conclusion

This paper presents a consolidity analysis for variable speed wind turbines (VSWTs) rotor examining their behaviour under the effect of the internal influences (blade pitch angle) and external influences (wind speed). The paper also shows how the VSWTs changes the consolidity pathway trajectory. The analysis study was applied for three case studies, first case as (theoretical analysis) from literature. Second and third case, practical data, obtained from the wind farms. The results obtained show that although the VSWTs are stable and controllable they are unconsolidated systems.

The maximum extracted mechanical power from the wind depends on the power coefficient of a wind turbine, which is function in both blade pitch angle and tip speed ratio. It has been shown that how the consolidity index and consolidity pathway trajectory of the VSWTs is changing with the blade pitch angle, tip speed ratio and wind speed. The proposed result illustrated how the formula of the power coefficient of wind turbine can play important role in the average consolidity index and consolidity pathway.

Wind energy conversion systems are complex systems in real life, which is influenced by many parameters that lead the consolidity index of the VSWTs to decrease with the increase of both wind speed and blade pitch angle; however, the consolidity pathway remains in the unconsolidated zone. This consolidity pathway gives deep view of the VSWTs consolidity index during its operation. Consolidity index thus must be taken in consideration while designing the wind turbines. In conclusion, the consolidity theorem is powerful, systematic tool for wind turbines manufactures that may consider avoiding the loss of operation of the VSWTs and their failures in the future.

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