A New Approach to Estimating Solar Radiation

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Abstract- Solar radiation on a region is one of the main parameters in deciding on the economy of conversion devices designed to utilize solar energy. Additionally, for solar energy investments, it is highly essential to predict the annual average solar radiation on the region. The literature indicates that the Ångström formula has, most of the time, been used for such estimations. However, in this paper, for the first time in literature, monthly average daily global radiation on the horizontal surface of Karaman region was calculated by means of spline interpolation, cubic spline regression, local polynomial regression, and Ångström methods, and the outcomes were compared. As a result, it has come out that the cubic spline method gives the best-fit values. Accordingly, it is concluded that the cubic spline model offers, statistically, better estimation results.

Keywords Ångström equation, non-parametric regression, parameter estimation, piecewise cubic spline regression, spline interpolation method, solar radiation.

1. Introduction

It is for sure that one of the crucial parameters in the design and operation of solar energy studies is the knowledge of solar radiation at a specific region [1]–[3]. In other words, solid knowledge of the solar resource at a site is useful for scientists, researchers, business people, investors, and so on [4]–[6]. Accordingly, the literature has several studies on the statistical analysis of solar radiation data. For instance, some researchers used fuzzy regression with support vector machine approach [7], some of them used deep learning model [8], [9], some of them used neural networks and used statistical indicators like R^2, RMSE, MBE and MAPE [10], some of them compared solar radiation data measured with satellite and ground station measurement [11], and some of them compare the Ångström model with some other models [12] Moreover, the literature review also indicates that different researchers tested the accuracy of the model by means of different statistical indicators and offer the third-order polynomial model [13] some researchers offer a quadratic model to estimate solar radiation [14], some of them use both the Ångström model and non-linear polynomial relations [15]. The Solar Energy Potential Atlas (SEPA) of Türkiye [16] which is useful for solar energy researchers indicates that the average annual total sunshine duration in Türkiye is 2,741 hours, the average annual total radiation is 1,527.46 kWh/m² and the average daily solar energy density is 4.18 kWh/m². In SEPA, the global radiation on the horizontal surface, in the general potential view for different months of the year, is the lowest in December with daily average radiation of 1.59 kWh/m²/day, and highest in June with 6.57 kWh/m²/day [17].

In recent years, Republic of Türkiye has placed great emphasis on renewable energy investments. Namely, Türkiye's installed solar-powered electricity capacity was 249 MW, which was 0.34% of the total installed electricity capacity in 2015, however, by the end of June 2022 it was 8,479 MW which was 8.35% of the total installed electricity capacity, and this ratio is increasing each year [17]. The installment of a power plant with a quite large capacity in Karapınar region, located about 100 kilometers from Karaman, is a clear indication of the growing interest in solar energy.

2. Method

In this part of the study, theoretical analysis of the Ångström formula, piecewise cubic interpolation method, local polynomial regression, and the statistical analysis of the data have been presented. However, the novelty of this study lies in the statistical analysis part which deals with Karaman data.

2.1. *Theoretical Analysis*

To estimate solar radiation, a number of empirical models are available in the literature. The Ångström-type regression equation is one of the most widely known, and is used as follows:

$$H = H_0\left(a + b\frac{t}{t_0}\right)$$

H is the monthly average daily global radiation, H_0 is the monthly average daily extraterrestrial radiation, t is the sunshine duration, t_0 is the possible maximum sunshine duration which is the time between atmospheric sunrise and sunset, a and b are empirical coefficients [18]–[20]. For a horizontal surface, monthly average daily extraterrestrial radiation can be calculated using the following equations:

$$H_0 = \frac{24}{\pi} G_0(Cos\varphi Cos\delta Sin\omega_s + \frac{\pi}{180}\omega_s Sin\varphi Sin\delta)$$

here,

$$G_{on} = G_{sc}\left[1 + 0{,}033\ Cos(\frac{360 n_{day}}{365})\right]$$

and G_{sc} is the solar constant, (1367 W/m²), φ is the latitude of the site, δ is the solar declination, ω_s is the main sunshine hour angle for the month, and n_{day}, is the number of the day of year, as of the 1st of January. The solar declination, main sunshine hour angle for the month, and highest possible sunshine duration (t_0) are as follows [20]:

$$\delta = 23.45 sin\left[\frac{360}{365}\ (284 + n_{day})\right]$$

$$\omega_s = cos^{-1}(-\ tan\ \varphi\ tan\ \delta)$$

$$t_0 = \frac{2}{15}\omega_s$$

Monthly average clearness index K_T is defined as [20]:

$$K_T = \frac{H}{H_0}$$

As already mentioned, Ångström-type equations have been used by several researchers to estimate the monthly average daily global radiation on a horizontal surface [21]–[23]. However, some researchers [24], [25] stated that the linear relationship was not valid for extremely large and small t and H values anymore. Instead, a more reliable and appropriate model was proposed as a non-linear relationship [24].

$$\eta = \eta_0 + \{a\xi^2 + b\xi\}^{1/2}$$

where

$$\eta = \frac{H}{H_0} \text{ and } \xi = \frac{t}{t_0}$$

a hyperbolic equation was also proposed as

$$\frac{(\xi + a)^2}{a^2} - \frac{(\xi + \xi_0)^2}{b^2} = 1$$

In calculating the average global daily solar radiation by means of Ångström's method, a functional dependence is assumed to be $\frac{H}{H_0}$ and $\frac{t}{t_0}$ [26]. These are linear equations in the form of

$$Y = a + bX$$

and

$$Y = \frac{H}{H_0} \text{ and } X = \frac{t}{t_0}.$$

The formula proposed by the researchers is the equation that meets the cubic spline conditions.

$$\frac{H}{H_0} = a + b\left(\frac{t}{t_0}\right) + c\left(\frac{t}{t_0}\right)^2 + d\left(\frac{t}{t_0}\right)^3$$

Here a, b, c, and d are the constants to be calculated.

2.2. *Local Polynomial Regression*

In non-parametric regression x and Y are the explanatory and dependent variables, respectively, and the purpose is to estimate the conditional expected value $E(Y|X = x) = m(x)$. Here, m(x) function is the conditional expected value of the random variable Y for a known x value of the random variable X. Accordingly, the non-parametric simple regression model can be;

$$Y_i = m(x_i) + \varepsilon_i\ , (i = 1, 2, 3, \dots, n) \qquad (1)$$

Here, m(.) denotes the unknown model function and ε_i's are random error term with 0 mean and σ^2 constant variance. Non-parametric simple regression is often called "dot plot smoothing" because the explanatory variable and dependent variable follow a smoothing path in the scatter plot. Local polynomial regression, a non-parametric regression sometimes named as smoothers in the literature, is also known as LOWESS (**LO**cally **WE**ighted **S**moothing **S**catterplot) which is an extension of "weighted moving average" concept used in time series analysis [27]. If each unknown function coefficient (β_j)s of the function $m(x_i)$ in equation (1) is considered as an unknown parameter,

$$m(x_i) = \sum_{j=0}^{p} \beta_j (x_i - x_0)^p$$

$$\min_{\beta} \left\{ \sum_{i=0}^{n} \left(Y_i - \sum_{j=0}^{p} \beta_j (x_i - x_0)^p \right)^2 K_h(x_i - x_0) \right\} \quad (2)$$

the solution to the regression model in equation (2) can be found by the weighted least squares method. Here K is the kernel function, and h controls the size of the approximate x_0 point. The choice of kernel function, calculation of h and R programming codes related to this topic are available in [28].

2.3. *Piecewise Cubic Spline Functions*

Spline interpolation has been of interest for the last few decades [29]–[31]. The literature has also some studies [2], [32] that statistically analyze solar radiation data using the spline interpolation method.

Fitting a polynomial curve for specific knots has important applications in both drawing and computer graphics. It is also important to draw a smooth curve through the knots. For that purpose, the spline function has been developed, and widely used in engineering.

Mathematically, on each sub-interval $[X_k, X_{k+1}]$, such cubic functions $S_k(X)$ can be created that the first and second-order derivatives of the resulting piecewise function $S(X)$ on the interval $[X_0, X_n]$ are continuous on the interval $[X_0, X_n]$. Thus, in cubic spline functions, the natural oscillation effect in higher-order interpolation polynomials is eliminated. Here, the first-order derivative is continuous, which means that the function has no sharp endpoints, and the second-order derivative is continuous, which means that the declination angle is defined at every point of the function [18], [33]. The gist of the cubic spline function is given in Figure 1.

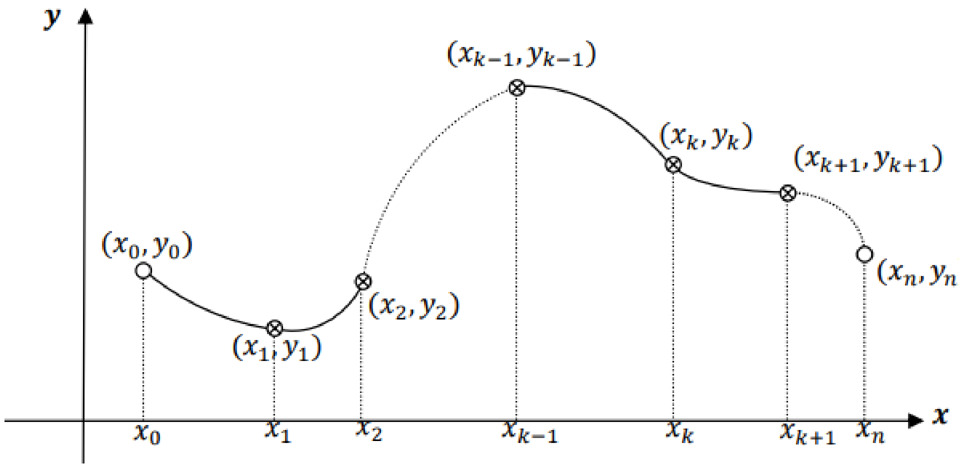

Fig. 1. $S(X)$Segmented polynomial consisting of $S_k(X)$ cubic interpolation polynomials

Given the data set $\{(x_k, y_k)\}$, k = 0, ..., n, where $x_0 < x_1 < \cdots < x_n$. If there are n cubic polynomials $S_k(X)$ meeting the following conditions, the piecewise function $S(X)$ is called a cubic spline function (Eq. 3). The function $S(X)$ has *4n* unknowns. To find these unknowns the following assumptions can be made:

(1) For $x \in [X_k, X_{k+1}]$ each S_k is polynomial
$$S(X) = S_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3, k = 0, 1, \ldots, n-1$$
is a cubic polynomial.

(2) The spline function passes through each knot. So,
$$S(X_k) = (y_k)\ k = 0, 1, \ldots, n$$

(3) Due to continuity, values of the function at interior knots are equal. So,
$$S_k(x_{k+1}) = S_{k+1}(x_{k+1})\ k = 0, 1, \ldots, n-2$$

(4) The first-order derivatives are equal at interior knots. So,
$$S_k(x_{k+1}) = S_{k+1}(x_{k+1})\ k = 0, 1, \ldots, n-2$$

(5) The second-order derivatives are equal at interior knots. So,
$$S"_k(x_{k+1}) = S"_{k+1}(x_{k+1}),\ k = 0, 1, \ldots, n-2$$

(6) Zero condition: The second-order derivatives are zero at the two ends. So,
$$S"(x_0) = S"(x_n) = 0$$

Although different assumptions for zero condition can be mentioned for a cubic spline function, the 6th assumption above is satisfactory, and a function with such natural boundaries is called a "natural cubic spline function". Further information on this is available in [31], [34].

2.4. *The Data and Statistical Analysis*

Knots are the points dividing the regimes and allowing for changes in the relationship between the dependent and independent variables [35]. In other words, a knot is the end of one interval and the beginning of another. Therefore, a knot is a place where the movement or direction of the function changes [36]. When the number and location of the knots are known, the parameter estimates of the regression equation can easily be done [37]. In this paper, the researchers made mathematical calculations using spline interpolation, cubic spline regression, local polynomial regression, quadratic spline, and classical Ångström methods. The calculations were made using Maple [38] for Spline interpolation, and R programming [39] for the remaining methods. The cubic spline regression model fitted to the $x_0, x_1, \ldots x_{k+1}$ intervals on the x-axis, where k is the number of knots, is written as follows [40]:

$$y = \sum_{j=0}^{n} B_{0j}\, x^j + \sum_{i=1}^{k} \sum_{j=0}^{n} B_{ij}(x - t_i)_+^j + \varepsilon \qquad (3)$$

In equation 3 above y is the continuous dependent variable which is $\frac{H}{H_0}$, and x is the explanatory variable which is $\frac{t}{t_0}$. t_i being the knot $(i = 1, \ldots, k)$ k is the set of knots formed by special x values (31, 60, 91, 121, 152, 182, 213, 244, 274, 305 and 335th days of the year). The data for the 1st of January is considered as the 366th day data. B's are the regression coefficients and ε is the regression error term [41] and in this paper $k = 11$ and $j = 3$.

In Türkiye, for years, solar radiation measurements were made using different actinographs at different stations of the Turkish State Meteorological Service [42], [43]. Global solar radiation, which is the sum of daily direct and diffuse radiation on a horizontal surface, is measured by means of a pyranometer, and the sunshine duration is measured by means of a "Heliograph".

Situated in the Central Anatolia Region, Türkiye, Karaman is located at 37º11' (N) latitude, 33º15' (E) longitude, and has a 1039-meter elevation. Known as an agricultural region, recently, Karaman has developed an agriculture-based industry thanks to the government incentives for the food industry. Additionally, in parallel with the industrial investments in the region, it has also received state-supported investments for alternative energy resources. Therefore, considering the industrial investments in the region, the importance of the energy to be used in the industry gets clearer.

In the literature, there are several statistical methods to evaluate the performance of estimation models. Among them, mean squared error (MSE), the root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and the coefficient of determination R^2 are commonly used [44] and each of them can be calculated using the following formulas;

$$MSE = \frac{1}{n}\sum [y_i - \hat{y}_i]^2$$

$$RMSE = \sqrt{\frac{1}{n}\sum [y_i - \hat{y}_i]^2}$$

$$MAE = \frac{1}{n}\sum |y_i - \hat{y}_i|$$

$$MAPE = \frac{100}{n}\sum_{i=1}^{n} \left|\frac{y_i - \hat{y}_i}{y_i}\right|$$

Here, y_i denotes the observed value and $\hat{y}_i$ denotes the estimated value.

In this paper, the researchers used the observation data available from The Turkish State Meteorological Service [45], which were recorded at the meteorological station in Karaman over a 19-year period, from 2000 to 2018. In the analysis, $\frac{t}{t_0}$ values are the independent variables, and observed solar radiation values are the dependent variables. The researchers made calculations taking the above-mentioned approaches into account, and the outputs are presented in Table 1.

Table 1. Comparison of model performance

Model	R^2	MSE	RMSE	MAE	MAPE
Spline Interpolation Method	0.87078	4.49812	2.12087	1.70904	10.89150
Local Polynomial Regression	0.95146	1.84575	1.35858	1.09173	7.72599
Cubic Spline Regression	**0.96087**	**1.78342**	**1.33544**	**1.06254**	**7.56462**
Ångström Metod	0.96032	1.80861	1.34484	1.07757	7.66594
Quadratic Spline Regression	0.95340	1.79944	1.34143	1.06602	7.59029

Lower MSE, RMSE, MAE and MAPE values, but higher R^2 values indicate better estimation performance. Taking Table 1 into account, it is obvious that the cubic spline model has the lowest MSE, RMSE, MAE and MAPE values; but the highest R^2 values. Accordingly, the cubic spline model offers statistically better estimation results.

The syntax used for computation in R programming for spline regression is;

```
library(splines)
cp=data$days[c(1, 32, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335)]
fit=lm(H ~ bs(days,knot = cp),data = data)
```

The syntax used for computation in Maple for spline regression is;

```
with(CurveFitting):
Spline([1, 32, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335, 366], [6.241, 10.915, 12.655, 22.137, 22.348, 19.535, 22.661,24.98,22.166,16.999,12.763, 9.01, 6.241], x, degree=3);
```

Starting from the 1st of January, the pre-determined number of days (1, 32, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335, and 366th days) were taken as the knots and the model was fitted using the spline interpolation, local polynomial regression, cubic spline regression, Ångström, and quadratic spline models. The researchers used the data for the 1st of January as the 366th day data. The graph of the models is given in Figure 2.

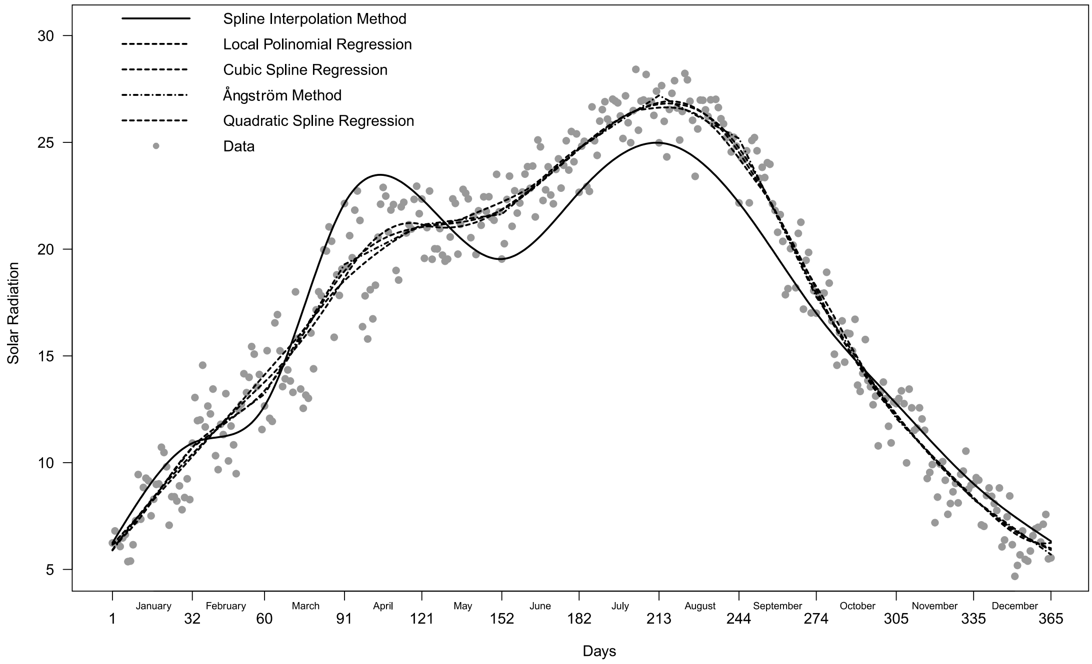

Fig. 2. Model graph

3. Conclusion

In today's world where seasonal changes are effective it is important for an energy entrepreneur to have the solar map of a region to invest in the region. In addition to the classical statistical models used in the literature for energy estimation for regional mapping, a more up-to-date method with less error is proposed in this paper.

The Ångström coefficients are of high importance and widely used for determining the solar energy potential of a region. Nevertheless, in this paper, the researchers put forward a new approach to analyzing solar radiation data in Karaman, Türkiye. In regions where solar power plants to be installed, one of the most important inputs for the investor is the insolation map, and the measurement of the radiation coming on the region for years. Moreover, not only the intensity of solar radiation but also a high clarity index makes the region attractive for investors.

In this paper, spline interpolation, local polynomial regression, cubic spline regression, quadratic spline regression, and Ångström models were compared to calculate solar radiation on the horizontal surface in Karaman where there are already large and small solar power plants, and new ones continue to be installed. The results indicate that cubic spline regression method has lower MAE, MAPE, RMSE and MSE values, yet higher R^2 values compared to the other methods. Since lower MAE, MAPE, RMSE, and MSE values but high R^2 values indicate a better-fit, it is concluded that among the models, the cubic spline model gives the best-fit values, and the model is statistically significant. As a result, it can be concluded that the cubic spline model gives statistically more consistent and accurate estimations.

Today, considering the climate crisis in the world, and the need to analyze the regions where solar energy investments will be made depending on the analysis of long-term data, it is thought that the researchers will frequently refer to the above-mentioned approaches and formulas. In brief, in this paper, the researchers put forth that using the cubic spline model gives statistically more consistent results with fewer errors in solar radiation estimation.

In today's world where the effects of seasonal changes are common, by means of this and similar studies, updating insolation maps will become more and more important for energy investors and industrialists not only today but also in the future. Furthermore, using the above-mentioned method and the data of the relevant region, and taking this research as a reference, solar radiation values can be calculated for specific regions in prospective studies.

Nomenclature

a, b, c, d	The coefficients
G_{on}	Extraterrestrial radiation (W/m²)
G_{sc}	The solar constant (1367 W/m²)
H	Monthly average daily global radiation on the horizontal surface (Mj/m²)
H_0	Monthly average daily extraterrestrial radiation on the horizontal surface (Mj/m²)

K_T	$\frac{H}{H_0}$ Monthly average daily clearness index
n_{day}	The number of the day (as of the 1st of January)
t	Meteorological sunshine duration (h)
t_0	Maximum sunshine duration (h)
ω_s	Main sunshine hour angle for the month (°)
cp	cut-off point
y	The observation value
$\hat{y}$	Estimated value using the model

Greek Letters

δ	*The solar declination (°)*
φ	*Angle of the latitude (°)*
η	$\frac{H}{H_0}$ *ratio*
ξ	$\frac{t}{t_0}$ *ratio*
φ	*Latitude of the site (°)*

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