Modelling of Optimal Tilt Angle for Solar Collectors Across Eight Indian Cities

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Abstract- In the present work, the optimum tilt angle for eight different cities across India is computed. The solar radiation received at various angles of tilt ranging from 1 to 90° is simulated and a yearly optimum tilt angle is determined for all the cities. From these results, a mathematical correlation to estimate the optimum tilt angle for any Indian city based on its latitude is proposed. The outcomes of this study will play a vital role in estimation and effective utilization of the freely available solar energy towards sustainable development of our nation.

Keywords- Latitude-based models; Optimum tilt angle; Yearly tilt angle; Solar collectors; Solar energy

1. Introduction

Extensive researches have been carried out in the recent years to obtain an in-depth knowledge of the available solar radiation in every part of the world so as to make this freely available energy a viable alternate source. Apart from the data available from meteorological observatories, simulation studies based on latitude-based solar models and Artificial Neural Network (ANN) have played a major role in enabling optimal design of solar-thermal systems. It has been established that proper orientation of solar collector is essential in order to maximize the solar energy utilization. Solar trackers have been developed for tracking the solar path and adjusting the collector tilt angle accordingly for the purpose of maximizing solar energy from the sun. Although tracking enables efficient means of solar radiation collection, it is costly and does not suit all solar applications. Thus, emphasis is laid on the optimal tilt angle of collectors so that they may be permanently fixed and still receive maximum radiation throughout the year.

2. Literature Background

Researchers have carried extensive studies to determine the optimum tilt angle to receive maximum solar radiation based on various parameters. Some of the key findings are discussed below :

Tian Pau Chang [1] performed an emprical study to determine the optimal tilt angle of the collectors in Taiwan. The estimations were based on three different radiation conditions namely extraterrestrial radiation, global radiation predicted by empirical model and ten-year observation data from 1990-1999. The monthly optimum tilt angles suitable for winter months was found to be nearly 50° and negative angle of -10° was estimated during summer. Jamil Ahmad & Tiwari [2] examined the theoretical aspects of choosing a tilt angle for solar flat plate collectors to receive maximum radiation at ten different stations in the world. The total solar radiation was estimated based on Isotropic radiation models like Badescu model (2002), Tian et al model (2001), Koronakis model (1986) & Liu Jordan model (1962) and also based on anisotropic models of Reindl et al (1990), Skartveit Olseth model (1986), Steven Unsworth model (1980) & Hay model (1979). The predicted values were checked with the solar radiation data obtained from India Meteorology Department, Pune, India for the period 1991-2001. It was observed that the monthly optimum tilt angles for December and January were 56° and 58° respectively, while the monthly optimum tilt angles is seen to be 0° during summer. The annual optimum angle is found to be equal to the latitude of the place and the loss of energy collected is 15% while using a yearly fixed angle of tilt instead of using monthly optimum tilts .

Al-Rawahi et al [3] modeled the hourly terrestrial radiation over horizontal and inclined surfaces for Muscat. The results were based on isotropic reflection of diffuse radiation and it was concluded that the optimum tilt during January was 40° towards south and for the summer season during June, the horizontal orientation is seen to absorb more solar radiation. The work by Akachukwu [4] reports the experimental studies of solar radiation measured by a solar radiometer mounted on a telescopic leg at an interval of 1° from the horizontal to vertical position. The optimum angle of tilt for Zaria was thus studied for all the months and an yearly average tilt was also determined. It was observed that the monthly optimum angle of inclination for trapping maximum solar radiation was 30° for the months January, June, November and December. On the other hand, the optimum angle of inclination was found to be 10° for the months March and September. With an yearly optimum angle of 22.5°, the average yearly solar radiation was found to increase by 4.23% as compared to the radiation obtained on a horizontal collector. Oko & Nnamchi [5] performed estimations to calculate the optimum tilt for the low latitudes of Nigeria ranging from 4.86 to 13.02°N and derived expressions for the monthly optimum tilt angle as a function of latitude for all the months. The results obtained from the monthly, seasonal and yearly analysis for eight cities showed that the monthly optimum tilt angles produce the largest annual cumulative insolation followed by the seasonal optimum tilt angles. The findings reported by Farzad et al [6] aims at determining the optimum tilt angle for south facing flat-plate collectors across 80 cities in Iran. The hourly solar radiation on the inclined surface calculated was checked with the measured data obtained from Islamic Republic of Iran. It was observed that the optimum tilt angle for some cities in specific summer days, May 1 to August 20 (n=120 to 220) was negative. Keshavarz et al [7] determined the daily, monthly, seasonal and yearly optimum tilts for 30 Iranian cities and provided atlases combining the data with Geographic Information System (GIS). The radiation estimated on a horizontal surface was compared with the solar data obtained from the Iranian Meteorological Organization for a period of 22 years from 1983-2005. The values of optimum slope angles for different months of a year for different cities was found to range from 46° to 59° during the month of January and negative angles in the range of -5.3° to 2.3° during the month of June. The investigations by Sekar et al [8] reports the thermal performance of a solar water heater inclined at angles between 0 to 30° at intervals of 10°. From a series of observations it was concluded that the optimum angle of solar water heaters in Coimbatore to be 10± 2° with the horizontal. Also, a 5 to 10° increase in angle of inclination for any particular latitude improved the thermal performance of the heater.

The mathematical modeling based on four year solar data in 35 different countries of the Mediterranean region by Hassane & Driss [9] resulted in a quadratic regression equation based on the site's latitude to calculate the optimum tilt angle. The obtained quadratic regression model satisfies all the statistical tests and assumptions and provides an accurate approximation of the annual optimal tilt angle for maximum solar radiation collection. Oloketuyi et al [10] estimated the optimum tilt angles for solar collectors in low-latitude tropical regions. The results obtained from the study shows an optimum tilt angle of Φ+25° for November, December and January ; Φ+15° for February, September and October ; Φ-15° for August ; Φ-25° for May, June, July and Φ for March and April where Φ is the latitude of the region. Bandyopadhyay et al [11] developed a mathematical model for the optimum orientation of a flat plate collector for two Indian cities Kolkata and New Delhi. The predicted values of optimum tilts were compared with that derived by similar studies by other researchers. An algorithm to calculate the optimum tilt angle of a solar panel was done by Emanuele [12] based on the global solar radiation data provided by Earth-based meteorological stations. The mathematical modeling was based on the maximization of the theoretical expression of the global solar irradiation impinging on an inclined surface, with respect to the slope and orientation of the panel and to the solar hour angle. The computed tilt angles were found to be significantly related to the latitude. Ajao et al [13] carried out experimental investigaions to estimate the optimal tilt angle for solar photovoltaic panel in Ilorin, Nigeria. The experiments were conducted by mounting a solar panel in the horizontal position and then increasing the orientation by 2° upto an inclination of 30°. It was found that the the solar panel gave maximum output at an angle of 22° (latiude + 13.5). Haixiang Zang et al [14] computed the the monthly, seasonal, and yearly optimum tilt angles for solar collectors in six different climatic zones of China using the measured weather data from China's meteorological stations over a 16-year period from 1994 to 2009. The yearly average optimum tilt angles for a south-facing solar collector were found to be 15°, 24.2° , 27.4° , 39.0° , 47.1° and 27.5° in Sanya (Φ=18°14'), Shanghai (Φ=31°24'), Zhengzhou (Φ=34°43'), Harbin (Φ=45°40'), Mohe (Φ=53°28'), and Lhasa (Φ=29°40'), respectively. It was also found that the yearly average of the optimum tilt angle is equal to the lattiude of the site.

It is seen from the above literature survey that the results of optimal tilt angles all over the world cannot be generalized and the results of study in a region cannot be applied to other regions as such, as the study results are region-specific except in few cases. Also, it is seen that, very few studies have been carried out in determining the optimal tilt angles for Indian cities and no correlation is available to estimate the optimum tilt for Indian cities. Hence, the present work aims to estimate the optimal tilt angles for Indian cities and to develop a mathematical expression suitable for all the cities in India which will be of great assistance while designing and installing solar-thermal equipments.

3. Estimation of Optimal Tilt for Indian Cities

India is situated in the northern hemisphere, north of the equator between 8°4' and 37°6' North latitude and 68°7' to 97°25' East longitude. The Indian cities chosen for the present study covering the entire landscape of India are : Trivandrum (8°48'N), Bangalore (12°97'N), Goa (15°29'N), Mumbai (19°07'N), Kolkata (22°57'N), Jaipur (26°91'N), New Delhi (26°91'N) and Srinagar (34°09'N). The anisotropic model based on Hay & Davis (1981) is used in the present analysis as it is the most suitable model to predict solar radiation in Indian cities **[15].**

A Matlab program is developed to estimate the daily average solar radiation on the tilted surface by varying the tilt angle (β) from 1 to 90° for the identified cities/latitudes (Φ) based on standard correlations **[15, 16].** The program is executed for each city for n=1 to 365 and the corresponding tilt angle (β_{opt}) which gives the maximum solar radiation for every month is noted. The same procedure is repeated for all the eight cities analyzed in this study. The methodology for the estimation of optimal tilt angle by varying the tilt from 1 to 90 ° is detailed below by a flowchart in Fig. 1.

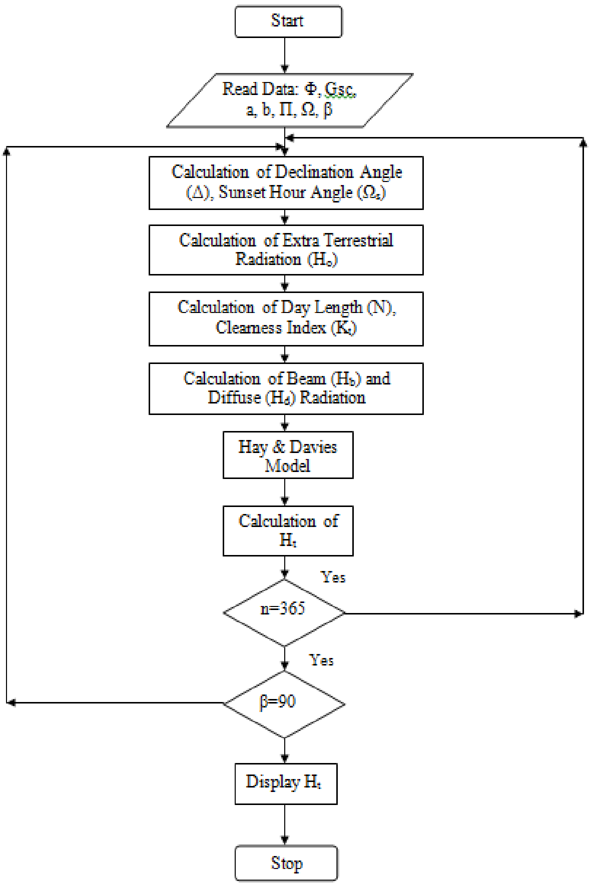

Fig.1 Flow chart for the estimation of optimal tilt angle

From the monthly average optimum tilt angles ($\beta_{M\text{-}opt}$), an yearly optimum tilt angle ($\beta_{Y\text{-}opt}$) is found out for each city so that the collector can be fixed for the entire year as against changing the orientation every month. The yearly optimum tilt obtained for each city is compared with that of NASA SSE and validated. An emprical relation to compute the yearly optimum tilt based on the lattiude is developed.

4. Results and Discussion

The Matlab program is executed from n = 1 to 30 for each month for an inclination of 0° to 90° and the optimum tilt angle that gives the maximum solar radiation is noted. The same procedure is repeated for the 8 chosen cities and the results are obtained as shown in Fig 2.

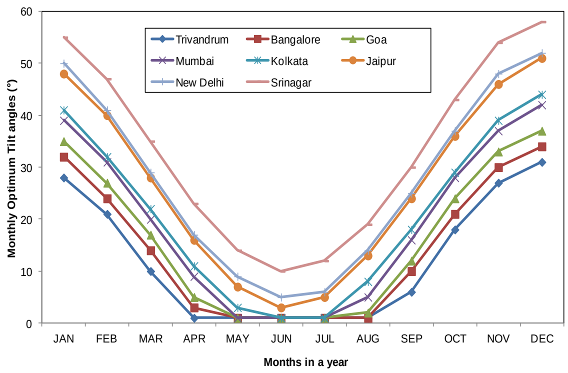

Fig.2 Comparision of monthly optimum tilts obtained for Indian cities

From Fig 2, it is evident that the monthly optimum tilt angles increase as the latitude increases. The monthly optimum tilt angles decrease from January to April and remain almost horizontal during the months of April to July and increase from August to December for all the cities covered in this study. It is clearly evident from the Fig that the optimum tilt angle for all the cities analyzed are nearly ϕ±20 for the winter months ie., January to March and October to December. The optimum tilt is inferred to be almost horizontal during the summer months ie., April to August for all the cities. However, in solar-based applications and installations, it is highly impossible for the user to vary the angle of tilt every month to obtain the maximum radiation. Hence, an optimum tilt of the receiver based on which the collector can be fixed for the whole year ($\beta_{Y\text{-}opt}$), with minimum loss of absorbed energy compared to changing the orientation every month would be a fitting solution. The average yearly tilt is obtained from the optimum monthly tilts using two methods : a) by computing the average of all the monthly optimum tilts ranging from January to December and b) by obtaining the RMS value of all the optimal tilts from January to December. This procedure is applied to all the cities and the yearly optimum tilt ($\beta_{Y\text{-}opt}$) for each city is compued. The yearly optimum tilts computed are compared with the yearly optimum tilt angles obtained from the Surface Meteorology, Atmospheric science data centre, NASA based on the the 22-year average monthly daily solar radiation (July 1983 through June 2005). The comparision of yearly average optimal tilt ($\beta_{Y\text{-}opt}$) based on average of monthly optimals, RMS average of monthly optimals and NASA SSE ($\beta_{Y\text{-}NASA}$) are shown in Fig 3.

The yearly optimum tilts estimated from the average of monthly optimal are seen to vary between 12.16° to 33.33° across Trivandrum and Srinagar. The yearly optimum tilt angles calculated using the RMS average of monthly optimal is found to vary between 16.78° to 37.43° across Trivandrum and Srinagar.

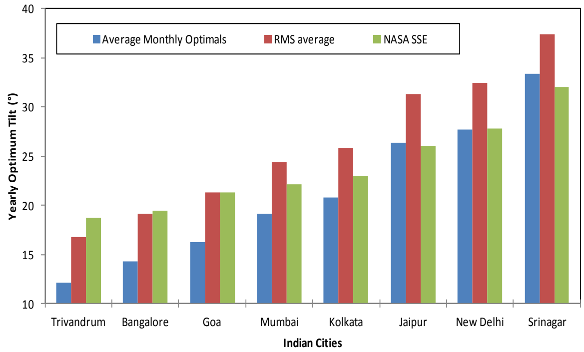

Fig.3 Comparision of yearly optimum tilts for Indian cities with NASA SSE

The variation in the yearly tilts between the average and RMS average is seen to be in the range of 4.1° to 4.62° across the Indian cities. The variation is higher in the southern cities and reduces towards the northern cities. The yearly optimum tilts as observed by NASA SSE for these Indian cities ranges from 18.7° to 32° for the cities considered in the present study. For the southern cities, the NASA SSE yearly tilt is close to the RMS average tilt whereas the monthly average tilt is close to NASA SSE tilt in the northern cities. In order to arrive at the best tilt among the three, the average solar radiation received by a receiver for an year is simulated for all the cities using these tilts. The comparision of the average solar radiation obtained using these tilts is shown in Fig. 4.

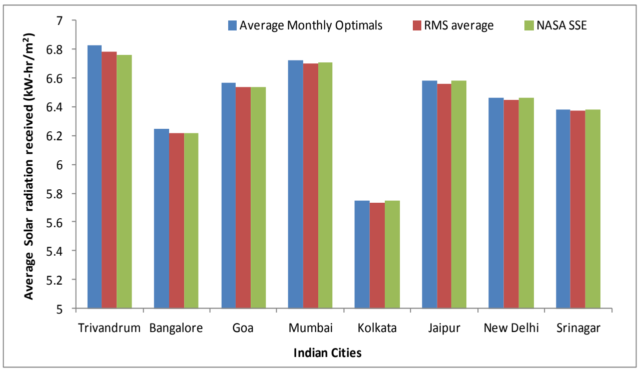

Fig.4 Comparision of maximum average solar radiation recieved using various tilts

From Fig. 4 it is seen that the average solar radiation obtained using the RMS average tilt matches closely with that of the NASA SSE. The average difference between the solar radiation obtained using RMS tilt and NASA SSE across all the cities is found to be 0.1 % only. Hence it may be concluded that the yearly optimum tilt angle ($\beta_{Y\text{-}opt}$) obtained from the RMS average of the monthly tilts can be used as the best tilt for the cities under study. Further, to obtain the optimum tilt angles of other Indian cities that are not covered in the study within the latitude of India (8°4'N to 37°6'N), a graph is plotted between the latitude of the city and the Yearly tilt based on

RMS average to obtain an empirical relation as shown in Fig 5.

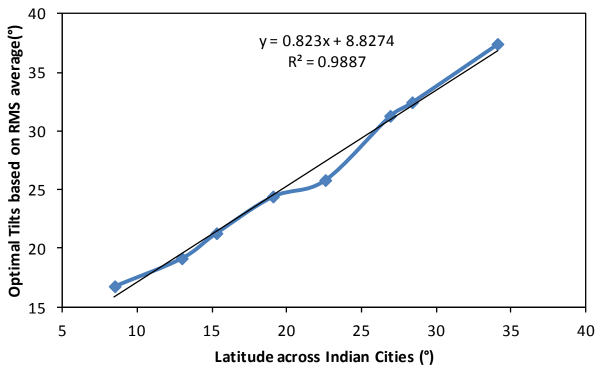

Fig. 5 Empirical relation between latitude and yearly optimal tilts for Indian cities

Thus, the yearly optimal tilt for any location in India can be estimated using the empirical relation obtained from the Figure 5. The relation obtained is shown by Equation (1).

Yearly Optimal Tilt (Y) = 0.823x + 8.8274
(1)

Where x = Latitude of the location

The yearly optimum tilt for the cities under study using the emprical relation proposed is shown in Table 1. A comparision of the new tilts proposed along with the tilts proposed by similar study is also presented in Table 1.

Table 1. Comparision of proposed yearly opimum tilts with other studies

City (Lattitude)	**Yearly Optimum tilts proposed by**					
	Jamil et al β = φ	**Eman uele β = φ**	**Haixi ang et al β = φ**	**Ajao et al β = φ+13.5**	**NASA SSE**	**Jims et al**
Trivandrum (8.48)	8.48	8.48	8.48	21.98	18.7	15.8
Bangalore (12.97)	12.97	12.97	12.97	26.47	19.4	19.5
Goa (15.29)	15.29	15.29	15.29	28.79	21.3	21.4
Mumbai (19.07)	19.07	19.07	19.07	32.57	22.1	24.52
Kolkatta (22.57)	22.57	22.57	22.57	36.07	23	27.4
Jaipur (26.91)	26.91	26.91	26.91	40.41	26	30.97
New Delhi (28.38)	28.38	28.38	28.38	41.88	27.8	32.18
Srinagar (34.09)	34.09	34.09	34.09	47.59	32	36.88

From the above table it is seen that the optimum tilt angles proposed by the present study are more accurate than the previous studies since the analysis has been obtained by a detailed sudy covering the entire spread of India for lattitudes ranging from 8°N to 35°N. Though the tilt angles are not close to those predicted by other researchers, they are pretty close to that of NASA SSE which has been derived from the observations ranging over 22 years. Hence the emprical relation proposed in this study can be used to obain the opimum tilt angle of the collector for any Indian city.

Conclusion

The estimation of the daily average solar radiation on the tilted surface by varying the tilt angle (β) from 1 to 90° for chosen cities/latitudes across eight Indian cities has been performed. The yearly optimum tilt angles obtained based on the RMS average of the monthly optimum tilts is found to match closely with the yearly optimum tilt angles of NASA SSE. Also, an emprical relation is obtained from the data generated to estimate the yearly opimum tilt angle at any location in India. This study will enable the more accurate estimation, design and effective utilization of the freely available solar energy towards sustainable development of our nation

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